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Sliding mode control for time delay systems based on output feedback sliding surface

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ABSTRACT: The problem of H_∞ output feedback sliding mode control for nonlinear systems with delay is considered. The systems have time varying uncertainty and external disturbance which satisfy the norm bounded condition. By LMI approach, a delay-dependent sufficient condition is given to design the H_∞ output sliding mode surface. Then the sliding mode controller satisfied the hitting condition is given to make the state of the states of systems hit the sliding mode surface in finite time. Finally, a numerical example is given to demonstrate the validity of the results.

Key Words: Time delay; H_∞ control; output feedback; sliding mode control

I. INTRODUCTION

Time delay systems often appear in various dynamical systems, such as communication systems, biological systems and so on. The existence of time delay often makes the system performance worse or even unstable. Therefore, in recent years, the study of time-delay systems has been widely concerned by many scholars, and a large number of system analysis and synthesis methods have emerged [1,2,3].

As we all know, the sliding mode control based on discontinuous control law is one of the most effective tools for the study of system stabilization. The design of fuzzy sliding mode controller for a class of uncertain time-delay systems is studied in [4]. Kown gives an improved delay dependent condition for robust control of uncertain time-delay systems [5]. Based on the LMI method, Chen considered the delay dependent exponential stability of a class of uncertain stochastic systems with multiple delays [6]. Xia and Qu combine LMI method to design robust sliding mode control for uncertain time-delay systems [7−8]. The output feedback sliding mode control of uncertain discrete time-delay systems is studied in [9]. However, the research on output feedback sliding mode control for time-delay systems has not been reported.

In this paper, the H_∞ output feedback sliding mode control of time-delay systems is studied. Firstly, the output feedback sliding surface of the system is designed by LMI method. Secondly, the sliding mode controller is designed to make the system state reach and stay on the sliding surface in finite time.

II. PROBLEM FORMULATION

Consider the following time-delay systems

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d) + Bu(t) + B_\omega \omega(t) \\ y(t) &= Cx(t) \\ x(t) &= \psi(t) \qquad -d \le t \le 0 \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the systems state vector, $u(t) \in R^m$ is the controlled input vector, $y(t) \in R^p$ is the systems output vector, d is the time delay. $\psi(t)$ is a real valued continuous initial function defined on $[-d, 0]$. $A \in R^{n\times n}, A_d \in R^{n\times n}, B \in R^{m\times n}, B_\omega \in R^{m\times n}$ and $C \in R^{p\times n}$ are constant matrices, B is column full rank, $\omega(t)$ is the external interference and satisfies:

$$\| \omega(t) \| \le \rho(t) \tag{2}$$

where $\rho(t)$ is a real valued continuous initial function defined on $[-d,0]$.
According to singular value decomposition of matrix B:

$$B=[U_1 \quad U_2]\begin{bmatrix}\Omega \\ 0\end{bmatrix}V^T$$

we construct the nonsingular transformation of system (1): $T=\begin{bmatrix}U_2^T \\ U_1^T\end{bmatrix}$, make

$$TB=\begin{bmatrix}0 \\ B_2\end{bmatrix};TB_\omega=\begin{bmatrix}B_{\omega 1} \\ B_{\omega 2}\end{bmatrix}.$$

Let $z(t)=Tx(t)$, we can obtain

$$\dot{z}(t)=\begin{bmatrix}\dot{z}_1(t) \\ \dot{z}_2(t)\end{bmatrix}=TAx(t)+TA_dx(t-d)+TBu(t)+TB_\omega\omega(t)$$

$$=TAT^{-1}z(t)+TA_dT^{-1}z(t-d)+TBu(t)+TB_\omega\omega(t)$$

i.e.

$$\begin{aligned}\dot{z}_1(t)&=U_2^TAU_2z_1(t)+U_2^TAU_1z_2(t)+U_2^TA_dU_2\\&\quad z_1(t-d)+U_2^TA_dU_1z_2(t-d)+B_{\omega 1}\omega(t)\\\dot{z}_2(t)&=U_1^TAU_2z_1(t)+U_1^TAU_1z_2(t)+U_1^TA_dU_2\\&\quad z_1(t-d)+U_1^TA_dU_1z_2(t-d)+B_2u(t)+B_{\omega 2}\omega(t)\end{aligned} \quad (3)$$

For the system (3), the output feedback sliding surface is selected as follows

$$\sigma(t)=Sy(t) \quad (4)$$

Let $\sigma(t)=Sy(t)=SCT^{-1}z(t)=SC[U_2 \quad U_1]z(t)=SCU_2z_1(t)+SCU_1z_2(t)=0$, and suppose matrix SCU_1 is nonsingular, we obtain

$$z_2(t)=-(SCU_1)^{-1}SCU_2z_1(t)=-Fz_1(t)$$

where $F=(SCU_1)^{-1}SCU_2$.

By substituting the above formula into the system (3), the sliding mode equation is obtained

$$\dot{z}_1(t)=\bar{A}z_1(t)+\bar{A}_dz_1(t-d)+B_{\omega 1}\omega(t) \quad (5)$$

where

$$\bar{A}=U_2^TA(U_2-U_1F)$$

$$\bar{A}_d=U_2^TA_d(U_2-U_1F)$$

III. MAIN RESULTS

Lemma 1[4] The LMI $\begin{bmatrix}Y(x) & W(x) \\ * & R(x)\end{bmatrix}>0$ is equivalent to

$$R(x)>0, Y(x)-W(x)R^{-1}(x)W^T(x)>0$$

where $Y(x)=Y^T(x), R(x)=R^T(x)$ depend on x.

Theorem1 if there exist $(n-m)\times(n-m)$ positive-definite matrices $\tilde{P},\tilde{Q},R$, matrices $X,\tilde{N}_1,\tilde{N}_2,\tilde{N}_3$, constants ρ_2,ρ_3 and $m\times(n-m)$ matrix Z such that the following matrix inequalities holds

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$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & -B_{\omega 1}X^T & d\tilde{N}_1 \\ * & \Sigma_{22} & \Sigma_{23} & -\rho_2 B_{\omega 1}X^T & d\tilde{N}_2 \\ * & * & \Sigma_{33} & -\rho_3 B_{\omega 1}X^T & d\tilde{N}_3 \\ * & * & * & -R & 0 \\ * & * & * & * & -d\tilde{Q} \end{bmatrix} < 0 \qquad (6)$$

where

$$\Sigma_{11} = \tilde{N}_1 + \tilde{N}_1^T - U_2^T A(U_2 X^T - U_1 Z) - (U_2 X^T - U_1 Z)^T A^T U_2$$

$$\Sigma_{12} = \tilde{N}_2^T - \tilde{N}_1 - U_2^T A_d (U_2 X^T - U_1 Z) - \rho_2 (U_2 X^T - U_1 Z)^T A^T U_2$$

$$\Sigma_{13} = \tilde{P} + \tilde{N}_3^T + X^T - \rho_3 (U_2 X^T - U_1 Z)^T A^T U_2$$

$$\Sigma_{22} = -\tilde{N}_2 - \tilde{N}_2^T - \rho_2 U_2^T A_d (U_2 X^T - U_1 Z) - \rho_2 (U_2 X^T - U_1 Z)^T A_d^T U_2$$

$$\Sigma_{23} = -\tilde{N}_2^T + \rho_2 X^T - \rho_3 (U_2 X^T - U_1 Z)^T A_d^T U_2$$

$$\Sigma_{33} = d\tilde{Q} + \rho_3 X^T + \rho_3 X$$

The sliding surface of the system is designed as

$$\sigma(t) = Sy(t)$$

where matrix S satisfies

$$SC(U_1 F - U_2) = 0, F = ZX^{-T}$$

then the sliding mode equation (5) is asymptotically stable, and the H_∞ performance index γ is the Singular value of $X^{-1}RX^{-T}$.

Proof The Lyapunov function is selected as follows:

$$V(t) = z_1^T(t)Pz_1(t) + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)dsd\theta$$

where P, Q are the undetermined positive definite matrices.Then, along the solution of system (5) we have

$$\begin{aligned} \dot{V}(t) = {} & 2z_1^T(t)P\dot{z}_1(t) + d\dot{z}_1^T(t)Q\dot{z}_1(t) - \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds + 2(z_1^T(t)N_1 + z_1^T(t-d)N_2 \\ & + z_1^T(t)N_3)(z_1(t) - z_1(t-d) - \int_{t-d}^{t} \dot{z}_1(s)ds) + 2(z_1^T(t)M_1 + z_1^T(t-d)M_2 \\ & + \dot{z}_1^T(t)M_3)(-\bar{A}z_1(t) - \bar{A}_d z_1(t-d) + \dot{z}_1(t)) \\ \le {} & 2z_1^T(t)P\dot{z}_1(t) + d\dot{z}_1^T(t)Q\dot{z}_1(t) - \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds + 2(z_1^T(t)N_1 + z_1^T(t-d)N_2 \\ & + z_1^T(t)N_3)(z_1(t) - z_1(t-d)) + 2(z_1^T(t)M_1 + z_1^T(t-d)M_2 \\ & + \dot{z}_1^T(t)M_3)(-\bar{A}z_1(t) - \bar{A}_d z_1(t-d) - B_{\omega 1}\omega(t) + \dot{z}_1(t)) + d(z_1^T(t)N_1 + z_1^T(t-d)N_2 \\ & + z_1^T(t)N_3)Q^{-1}(z_1^T(t)N_1 + z_1^T(t-d)N_2 + z_1^T(t)N_3)^T + \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds \\ & - \gamma^2\omega^T(t)\omega(t) + \gamma^2\omega^T(t)\omega(t) \\ = {} & \xi^T(t)\Xi\xi(t) + \gamma^2\omega^T(t)\omega(t) \end{aligned}$$

where $N_1, N_2, N_3, M_1, M_2, M_3$ are undetermined constant matrices with appropriate dimensions

$$\xi(t)=\begin{bmatrix} z_1^T(t) & z_1^T(t-d) & \dot{z}_1^T(t) & \omega^T(t)\end{bmatrix}^T$$

$$\Xi=\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & -M_1B_{\omega 1} \\ * & \Xi_{22} & \Xi_{23} & -M_2B_{\omega 1} \\ * & * & \Xi_{33} & -M_3B_{\omega 1} \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Xi_{11}=N_1+N_1^T-M_1\bar{A}-\bar{A}^TM_1^T+dN_1Q^{-1}N_1^T$$

$$\Xi_{12}=N_2^T-N_1-\bar{A}^TM_2^T-M_1\bar{A}_d+dN_1Q^{-1}N_2^T$$

$$\Xi_{13}=P+N_3^T-\bar{A}^TM_3^T+M_1+dN_1Q^{-1}N_3^T$$

$$\Xi_{22}=-N_2-N_2^T-M_2\bar{A}_d-\bar{A}_d^TM_2^T+dN_2Q^{-1}N_2^T$$

$$\Xi_{23}=-N_3^T-\bar{A}_d^TM_3^T+M_2+dN_2Q^{-1}N_3^T$$

$$\Xi_{33}=dQ+M_3+M_3^T+dN_3Q^{-1}N_3^T$$

The Inequality $\Xi<0$ （7）
can be rewritten as

$$\Xi=\Theta+d\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ 0 \end{bmatrix}Q^{-1}\begin{bmatrix} N_1^T & N_2^T & N_3^T & 0\end{bmatrix}<0$$

where

$$\Theta=\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & -M_1B_{\omega 1} \\ * & \Theta_{22} & \Theta_{23} & -M_2B_{\omega 1} \\ * & * & \Theta_{33} & -M_3B_{\omega 1} \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Theta_{11}=N_1+N_1^T-M_1U_2^TA(U_2-U_1F)-(U_2-U_1F)^TA^TU_2M_1^T$$

$$\Theta_{12}=N_2^T-N_1-M_1U_2^TA_d(U_2-U_1F)-(U_2-U_1F)^TA^TU_2M_2^T$$

$$\Theta_{13}=P+N_3^T+M_1-(U_2-U_1F)^TA^TU_2M_3^T$$

$$\Theta_{22}=-N_2-N_2^T-M_2U_2^TA_d(U_2-U_1F)-(U_2-U_1F)^TA_d^TU_2M_2^T$$

$$\Theta_{23}=-N_3^T+M_2-(U_2-U_1F)^TA_d^TU_2M_3^T$$

$$\Theta_{33}=dQ+M_3+M_3^T$$

From lemma 1, we can get that matrix inequality (7) is equivalent to

$$\Delta=\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & -M_1B_{\omega 1} & dN_1 \\ * & \Delta_{22} & \Delta_{23} & -M_2B_{\omega 1} & dN_1 \\ * & * & \Delta_{33} & -M_3B_{\omega 1} & dN_1 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -dQ \end{bmatrix}<0 \quad (8)$$

$$\Delta_{11} = N_1 + N_1^T - M_1 U_2^T A(U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_1^T$$
$$\Delta_{12} = N_2^T - N_1 - M_1 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_2^T$$
$$\Delta_{13} = P + N_3^T + M_1 - (U_2 - U_1 F)^T A^T U_2 M_3^T$$
$$\Delta_{22} = -N_2 - N_2^T - M_2 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A_d^T U_2 M_2^T$$
$$\Delta_{23} = -N_3^T + M_2 - (U_2 - U_1 F)^T A_d^T U_2 M_3^T$$
$$\Delta_{33} = dQ + M_3 + M_3^T$$

For simplicity, suppose $M_1 = M_0, M_2 = \rho_2 M_0, M_3 = \rho_3 M_0$, where ρ_2, ρ_3 are undetermined constants. On both sides of inequality (8), multiply left by right and multiply by $diag\{M_0^{-1}, M_0^{-1}, M_0^{-1}, M_0^{-1}, M_0^{-1}, I\}$, and give some variable substitutions as $X = M_0^{-1}, Z = FX^T, \tilde{P} = XPX^T, \tilde{Q} = XQX^T, R = \gamma^2 XX^T$, the matrix inequality (8) is equivalent to (6).

With the matrix inequality（6）, we can obtain

$$\dot{V}(t) \le -\xi^T(t)\Xi\xi(t) + \gamma^2\omega^T(t)\omega(t)$$

By integrating the two sides of the above inequality

$$V(t) - V(t_0) \le -\int_{t_0}^{t} \xi^T(s)\Xi\xi(s)ds + \int_{t_0}^{t} \gamma^2\omega^T(s)\omega(s)ds$$

let $t \to 0$, we obtain

$$-\lambda_{\min}(\Xi)\int_{t_0}^{t} z_1^T(s)z_1(s)ds \le -\lambda_{\min}(\Xi)\int_{t_0}^{t} \xi^T(s)\xi(s)ds \le \gamma^2\int_{t_0}^{t} \omega^T(s)\omega(s)ds$$

therefore

$$\| z_1(t) \|_2 \le \frac{\gamma}{\sqrt{-\lambda_{\min}(\Xi)}} \| \omega(t) \|_2$$

If $\omega(t) = 0$, we can obtain $\dot{V}(t) < 0$. The sliding mode equation is asymptotically stable.

Theorem2 For the time delay systems（1）, selecting the sliding mode control

$$u(t) = -(SCB)^{-1}[SCAx(t) + SCA_d x(t-d) + \frac{\| SC \| \sigma(t)}{\| \sigma(t) \|} \| B_\omega \| \rho(t) + k\sigma(t) + \varepsilon sign\sigma(t)] \tag{9}$$

where $k > 0, \varepsilon > 0$ are constants, then the system states arrives the output sliding surface (4) in a finite time.

Proof Along the solution of system (1), we have

$$\begin{aligned} \sigma^T(t)\dot{\sigma}(t) &= \sigma^T(t)SCAx(t) + \sigma^T(t)SCA_d x(t-d) + \sigma^T(t)SCB_\omega\omega(t) \\ &\quad - \sigma^T(t)SCAx(t) - \sigma^T(t)SCA_d x(t-d) - \frac{\sigma^T(t)\| SC \| \sigma(t)}{\| \sigma(t) \|} \| B_\omega \| \rho(t) \\ &\quad - \sigma^T(t)k\sigma(t) - \sigma^T(t)\varepsilon sign\sigma(t) \\ &\le -\sigma^T(t)k\sigma(t) - \sigma^T(t)\varepsilon sign\sigma(t) < 0 \end{aligned} \tag{10}$$

According to the controller (9) and the equation (10), the arrival condition is established.

IV. CONSLUSION

In this paper, the output feedback sliding mode control for time-delay systems is considered. The output feedback sliding mode surface of the system is designed by Using LMI method, and the sliding mode controller of the system is designed on this basis, so that the system state can reach the system state in finite time.

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